lower than those predicted by the stress function approach, which is in agreement with previous observations.[4,6] It is possible to solve one of the nonlinear governing equations exactly in the stress function approach, whereas the Berger approach requires much less effort in the solution procedure.

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Structural Optimization by Mathematical Programming Methods

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Introduction

A COMPREHENSIVE study of mathematical programming methods as they apply to structural (mechanical) system design optimization has been completed recently under an NSF grant.[1] The major objective of the study was to develop a unified viewpoint of the modern mathematical programming (MP) methods as they apply to such engineering design problems. Each method is analyzed to determine its strengths and weaknesses. The methods analyzed are taken from the existing MP literature and, therefore, are not new. What is new is the analytical and numerical comparison of the methods, unified viewpoint, and development of a basis for studying methods as they apply to the engineering design environment. No "best method" is recommended since the meaning of "best" depends on an individual's viewpoint.

Very few comprehensive studies on comparison of methods for optimal design have been conducted. Some studies have been conducted in the MP literature.[1] However, those studies have the following limitations: 1) only small-scale problems are considered, 2) analytical aspects are not considered, 3) functions in the problem depend explicitly on design variables as opposed to implicit functions in structural design problems, and 4) problems with multiple local minima are not considered. The present study is conducted to overcome these limitations with a particular regard to structural design problems. The purpose of this Note is to present the findings of this study.

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Methods Considered

Only gradient-based methods are studied because quantities such as stress and displacement which enter into the structural optimization problem are usually continuously differentiable functions of design variables. Methods such as dymamic programming, geometric programming, physically motivated optimality criteria, and fully stressed design are not included in the study, since they are not suitable for general design applications. Also, methods requiring substantial interaction by the designer during the iterative process are not included in the study. While these methods are especially effective in certain situations, they are not general-purpose enough to be combined with analysis codes to form design systems.

Mathematical programming methods are based on solving the Kuhn-Tucker optimality conditions. They are classified into two categories: primal and transformation methods. The primal methods studied in this work are: 1) recursive quadratic programming (RQP) methods; 2) gradient projection methods; 3) reduced gradient methods; 4) method of Bard and Greenstadt; 5) feasible directions methods; 6) optimality criteria methods; 7) sequential linear programming (LP) methods; 8) projection methods; and 9) methods which solve nonlinear reduced problems. Transformation methods are conceptually different from primal methods. These methods transform the original constrained problem into a sequence of unconstrained problems. Within this category, the methods considered in the study are: 1) sequential unconstrained minimization techniques (SUMT) (penalty and barrier functions), and 2) multiplier (or augmented Lagrangian) methods.

Basis of Comparison

A basis for comparison of methods has been developed by considering features of the design problem and global convergence aspects. The basis is used to meet objectives of the study and consists of the following considerations.

1) Special structure possessed by a method. An essential difference between the structural optimization problem and the mathematical programming problem is that the functions in the former are implicitly dependent on design variables. The question therefore is whether or not there are methods whose structure is especially suited for handling such implicit functions.

2) Geometrical aspects. Almost all methods update the current design using an iterative process consisting of direction finding and step size determination problems. The geometrical significance of the direction vector for each method should be studied with a view toward bringing out similarities and differences between the methods.

3) Global convergence. The global convergence is a very desirable property for a method to possess. A method is said to be globally convergent if it converges to a solution from any starting design. Global convergence is an indication of reliability of a method. We need to examine whether the method possesses global convergence, and, if it does, what is its computational cost. Specifically, whether or not global convergence is achieved using an "active-set" strategy.

4) Active-Set strategy. In general, the direction vector at the current design point is determined using cost and constraint functions and their gradients. By an active-set strategy it is meant that, to determine the direction vector, only a subset of all the constraints is used at any iteration. Usually, the subset consists of only those constraints that are nearly satisfied or violated. The importance of using an active set is the savings in computation when the gradients of only a few (implicit) constraint functions have to be evaluated at each iteration.

5) Superlinear convergence. Superlinear convergence of an algorithm implies that the sequence of designs generated by it moves toward the solution point at a faster rate. Superlinear convergence is a desirable feature of any algorithm. It is possible to obtain superlinear convergence in some methods if a matrix containing curvature information of the cost and constraint functions is used. Specifically, the matrix is taken to be a positive-definite approximation to the Hessian of the Lagrangian function for the problem. The matrix at the kth step is usually estimated using quasi-Newton updates whereby information is accumulated from the preceding steps using only first derivatives. In connection with using such a matrix in various methods, the questions examined are: a) Is such a matrix used in the method? If not, whether or not it is possible to introduce the matrix. b) Whether or not any proofs of superlinear convergence have been presented associated with the method.

6) Computational aspects. Some aspects, such as amount of computation involved per iteration and the number of "optimization parameters" that have to be selected before executing the algorithm, are also considered. Clearly, if there are many parameters to be selected, then it is difficult for a general user to use the method. A computer code based on such a method will not be user-oriented.

Numerical Results

Based on the theoretical study, some promising algorithms are selected for numerical investigation. The primal methods selected are RQP algorithm of Pshenichny,[1] two algorithms based on a feasible directions method, and a gradient projection method. Among the transformation methods, the SUMT based on the exterior penalty function and three algorithms based on the multiplier methods are selected. Six codes based on these algorithms are developed. Two codes, OPTDYN and CONMIN, that are based on the method of feasible directions are acquired and used. Twelve structural design problems are selected. The number of design variables range from 3 to 47, and the number of constraints range from 3 to 252. All of the codes are run for each problem starting from the same design. Evaluation criteria to analyze test results are

1) Accuracy: The value of the cost function at the final design is compared.

2) Reliability: A reliable code is expected to give feasible solutions. A code may give accurate solutions for some problems but may fail to obtain feasible solution on others. Such a code is labeled as unreliable.

3) Efficiency: Total computing time and the number of calls to the function and gradient evaluation routines are used as a measure of the efficiency of a code.

The three criteria are not combined in any manner. A code is simply labeled as accurate, reliable, and/or efficient.

Discussion and Conclusions

It is found that many calculations in primal methods are similar. In particular, the direction-finding map is decomposed into two vectors that are orthogonal to each other. One vector is the projected cost gradient on to the constraint hyperplane and the other is normal to it. The first vector is responsible for reduction in the cost function and the second is responsible for constraint correction. Even the direction-finding map in multiplier methods is interpreted in terms of these vectors, thus unifying the two conceptually different approaches.

It is concluded that the gradient projection methods, method of Bard and Greenstadt, sequential LP, optimality criteria, and methods with nonlinear reduced problems are not globally convergent. Also, generalized reduced gradient methods become identical to the gradient projection methods since an active-set strategy must be used for structural optimization. Therefore, the two methods need not be distinguished in the context of structural optimization. In addition to not being globally convergent, these methods possess other drawbacks. Gradient projection methods ensure progress by taking a set of projection and correction steps to reduce the cost function from one feasible design to another. This process, however, involves orthogonal iterations that are tedious and inefficient. In addition, the very foundation on which gradient projection methods were developed is questionable for structural optimization. The methods were developed with the idea of determining a direction vector without having to solve a linear or quadratic program at each step, as is done RQP and feasible direction methods. Thus, in gradient projection methods, one seeks a direction that is not quite as good but easier to compute. Such an approach is questionable in structural optimization since the time taken to compute functions and gradients is generally much larger than that required to solve a linear or quadratic program.

Sequential LP methods, apart from not being globally convergent, possess major difficulties in determining bounds on the change in design. With a little extra effort, it is possible to use RQP methods that are free from these problems in addition to being globally and superlinearly convergent.

It is found that there are feasible direction methods that are globally convergent and use active-set strategy. These methods do not have superlinear convergence. However, a way of achieving superlinear convergence is presented. They also have two other drawbacks. The first one involves proper selection of certain parameters (which includes pushoff factors) for reliability and efficiency of the method. These parameters are difficult to estimate for different problems. The second drawback concerns problems with equality constraints. For such problems, it is difficult to compute feasible directions efficiently.

Among RQP methods, Pshenichny's method[1] is preferable to the other primal methods discussed above. It is globally convergent, uses active-set strategy, can be extended to have superlinear convergence, and maintains a descent property based on a relatively simple logic. The step size is computed using a descent function which involves a scalar r. The scalar serves as a weighting factor between the cost function and constraint violation in defining the descent function. If r becomes too large, the step size becomes too small which results in slower convergence. Ways of keeping r small and still being able to prove global convergence is a problem that has yet to be solved.

Transformation methods are philosophically different from primal methods. In primal methods attention is focused on each constraint function and its gradient. In transformation methods, on the other hand, the cost and constraint are collapsed into one function, the "transformation function." Sequential unconstrained miniimization of this function is then performed to obtain a solution. It turns out, that for structural design problems, the gradient of the transformation function can be calculated without calculating gradients of individual constraint functions. Therefore, transformation methods are able to exploit the implicit nature of the functions in the engineering design problems. Transformation methods have certain other strong points. They are globally convergent and use an active-set strategy. Step size selection is not a source of difficulty because the goal of minimizing an unconstrained function is well defined. Most important, a definite way of forcing global convergence exists through appropriate increases in the penalty parameters. This is why transformation methods are more reliable and robust than most primal methods. However, a difficulty in transformation methods is the amount of computation involved in performing the sequence of unconstrained minimizations. Thus, while transformation methods are very reliable, they are not usually very efficient.

Numerical investigation of various methods reveals that primal methods generally are more efficient as compared to transformation methods. The main reason for this inefficiency is the requirement of exact minimization of the transformation function at each iteration. The methods are quite robust as is the RQP method of Pshenichny[1] (a primal method). The RQP method converges to a local minimum point that is closest to the starting point. This motivates the development of hybrid methods where a large step algorithm may be used in the beginning to skip over certain local minima. Hybrid methods also are motivated by the desire to combine efficiency with reliability.

It is concluded that results from the study should form a basis for future developments in computational methods for optimum structural design.

Acknowledgment

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Supersonic Flutter of Short Panels on an Elastic Foundation

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Introduction

SUPERSONIC flutter of panels is the self-excited oscillation of the external skin of a flight vehicle exposed to an airflow along its surface. Reference 1 gives an excellent review of this topic. In Refs. 2-4 the finite element method has been successfully used to investigate the flutter of panels. The effects of shear deformation and rotatory inertia on the flutter of panels has been studied in Ref. 5.

In this Note the effect of an elastic foundation on the supersonic flutter of short panels is studied using the finite element method. The formulation used takes care of the effects of shear deformation and rotatory inertia. Flutter parameters are obtained for simply supported, simply supported and clamped, and clamped panels and the results are presented in tabular form.

Finite Element Formulation

The panel (Fig. 1) is represented by a flat plate of unit width in bending. The supersonic airstream flows over the upper surface of the plate in the positive X-direction and the bottom surface is supported on an elastic foundation.

The matrix equation governing the motion of the panel is

$$[K]\{q\}+\lambda[A]\{q\}+\gamma[F]\{q\}-\Omega\,[M]\{q\}=0 \tag{1}$$

where $[K]$ is the stiffness matrix including shear deformation, $[A]$ the aerodynamic matrix, $[F]$ the foundation matrix, $[M]$ the mass matrix including rotatory inertia, λ the dynamic pressure parameter, γ the foundation parameter, Ω the nondimensional eigenvalue parameter, and $\{q\}$ the eigenvector.

The nondimensional quantities λ, γ, and Ω are defined as

$$\lambda=2qL^3/D(M_\infty^2-1)^{1/2} \tag{2}$$

where q is the dynamic pressure, L the length of the panel, D the panel flexural rigidity, and M_∞ the Mach number;

$$\gamma=kL^4/D \tag{3}$$

where k is the foundation modulus per unit area; and

$$\Omega=\frac{M_\infty^2-2}{M_\infty^2-1}\frac{L}{U}\nu-\frac{mL^4}{D}\nu^2 \tag{4}$$

where U is the flow velocity, ν the flexural frequency, and m the mass per unit area.

The matrices $[K]$, $[M]$, and $[A]$ are assembled from the element matrices $[k]$ and $[m]$ taken from Ref. 6 and $[a]$ derived following the method proposed by Olson.[3,4]

Table 1 Values of λ_{cr} and Ω_{cr} for a simply supported panel

L/t	γ	λ_{cr}	Ω_{cr}
50.0	0.0	342.0	1047
	1.0	342.0	1048
	10.0	342.0	1057
	100.0	342.0	1147
	1000.0	342.0	2047
25.0	0.0	338.5	1036
	1.0	338.5	1037
	10.0	338.5	1046
	100.0	338.4	1135
	1000.0	337.4	2032
10.0	0.0	315.0	962.9
	1.0	315.0	963.9
	10.0	315.0	972.9
	100.0	314.3	1060
	1000.0	308.1	1934
5.0	0.0	248.4	755.1
	1.0	248.4	756.1
	10.0	248.0	763.9
	100.0	245.6	843.8
	100.0	245.6	843.8
	1000.0	220.5	1642

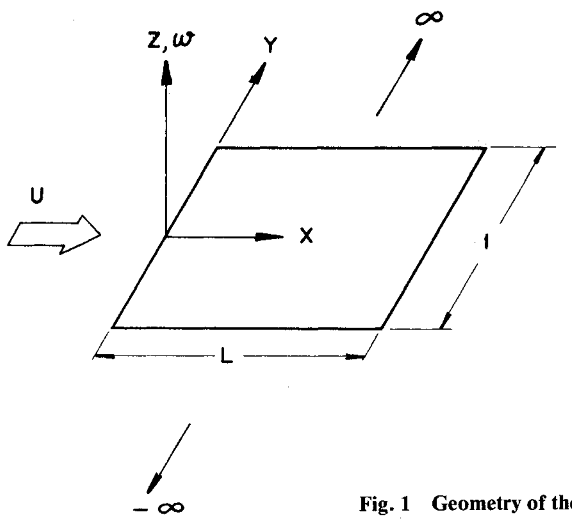

Fig. 1 Geometry of the panel.

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